

## FREQUENCY ANALYSIS OF NONLINEAR SHEAR WALL MODEL UNDER SEISMIC LOADING

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### Abstract

*The paper deals with dynamic analysis of reinforced shear wall under seismic loading. The shear walls of Kashiwazaki-Kariwa nuclear power plant structures reported no significant structural damage during 2007 Chuetsu earthquake despite of the fact, that the response spectra exceeded the design spectra of bearing structures. That was the reason for research of applied shear wall. The IRIS research project on industrial safety assessment and management included the cyclic testing of full-scale reinforced concrete walls. The results of the laboratory tests were used to analyze the nonlinear behavior of reinforced concrete wall under seismic excitation. The hysteresis (force-displacement relation) has been used for numerical SDOF model design. The calibrated model was used for earthquake simulations. Data of a set of strong motion records with different amplitudes, acceleration/velocity ratios and frequency content was selected to model excitation. The frequency analysis of the response signal was carried out with wavelet transformation. The frequency content at a certain time during the earthquake excitation was obtained. The responses of inelastic structure for different excitations were compared and analyzed in frequency domain.*

**Keywords:** Shear Wall, FE Model, Structural Dynamics, Earthquake Engineering.

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## 1 INTRODUCTION

Even though most of the surprises that earthquakes bring us are negative, examples on the positive note do exist. The 2007 Chūetsu offshore earthquake had an epicenter just 19km from the location of the Kashiwazaki-Kariwa nuclear power plant built in the 1980's. The earthquake shook the plant beyond design basis. The response spectra of the Chūetsu earthquake surpassed the design spectra for the investigated structures. The earthquake-proofing no significant structural damage was reported [1]. The power plant was shut down due to technological damage only. This was the incentive to delve deeper into the dynamic behavior of shear walls which are the structural elements that provide the necessary integrity to nuclear structures. It was also the reason to include base research in the form of shear wall testing in the IRIS research project on industrial safety assessment and management [2].

## 2 EXPERIMENTAL TESTING

The novel experimental testing was carried out (reported in Bekö et al. [3]). During cyclic testing (Fig. 1) of full-scale reinforced concrete low-rise shear wall specimen the pure shear was simulated. The obtained data was analyzed and well-defined hysteresis curve (Fig.2) was obtained.

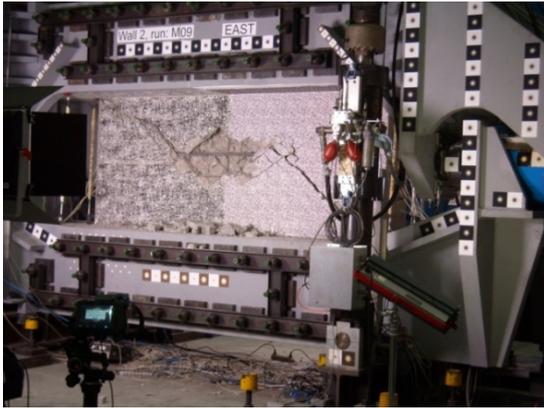


Figure 1: Reinforced concrete shear wall test.

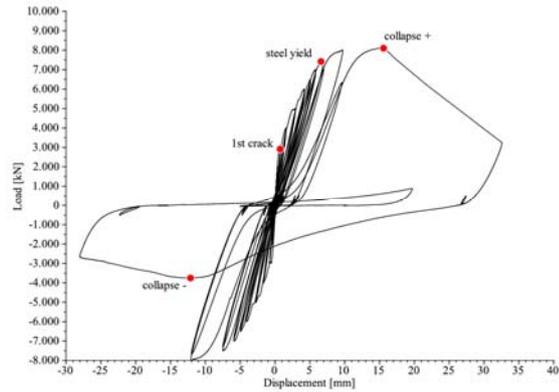


Figure 2: Test hysteresis (force vs displacement).

## 3 MODELING

A nonlinear mathematical model was created which is capable of simulating the hysteresis of the tested shear walls. The analytical model is a single DOF model of the Takeda family. Litton's [4] simplification of the Takeda model [5] was used as a base for the development of a dedicated model with stiffness degradation and pinching. The model is piecewise linear hysteresis model with a quadrilateral backbone curve and unsymmetrical positive and negative quadrants. Two separate sets of governing parameters were adopted for the positive and negative quadrants each with four variables  $\alpha, \beta, \gamma, \delta$ . The governing equations of the model are defined as follows (for definition of the symbols see Fig. 3)

$$- \text{ unloading stiffness } k_u = k_0 \left( \frac{u_{\max}}{u_y} \right)^{-\alpha} \quad (1)$$

$$- \text{ reloading point } Y = [u^+, f^+]; \quad u^+ = \frac{u_{\max}}{u_y} - \beta \frac{u_{\max} - u_y}{u_y}, \quad f^+ = f_y + k_t(u^+ - u_y) \quad (2)$$

- pinching stiffness  $k_s = \frac{f_{max}}{u_{max} - u_q} \left( \frac{u_{min}}{u_y} \right)^{-\gamma}$  (3)

- pinching point  $P = [u_p, f_p]$ ;  $u_p = \frac{-k_s u_q + k_u u_{max}}{k_u - k_s} \delta$ ,  $f_p = k_s (u_p - u_q)$  (4)

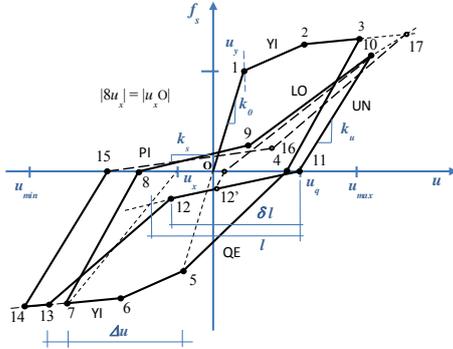


Figure 3: Takeda-Litton's model

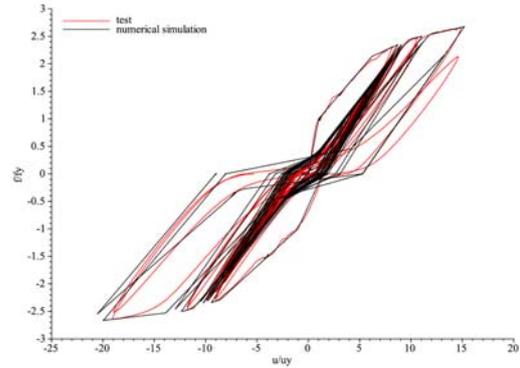


Figure 4: Test (red) and simulation (black) hysteresis

The verification of the model is shown in Fig. 4. The numerical investigations were hinged on an analytic model developed on the basis of laboratory tests. The model was used to analyze the nonlinear behavior of a single RC shear wall under seismic excitation. Various aspects of this behavior were studied using this model.

#### 4 TIME-FREQUENCY ANALYSIS

The calibrated model was used for earthquake simulations of an imaginary shear wall that is analogous to the test. Nonlinear response spectra were derived for a selected set of earthquakes. The earthquake selection was based on the following considerations. It was required to have a selection of strong motions with different A/V (maximum acceleration to maximum velocity) ratios. At the same time we wanted to have different frequency content, varying duration and overall amplitudes of the ground motions. This has led to a set of well-studied earthquakes summarized in in Table 1.

Earthquake	Date	Mw	Site	E.Dist	A (g)	V(m/s)	A/V
Central Honshu Japan	Feb. 26 1971	5.5	Yoneyama Bridge	27	0.151	0.060	2.52
San Fernando California	Feb. 9 1971	6.4	Pacomia Dam	4	1.076	0.577	1.87
Imperial Valley California	May 18 1940	6.6	El Centro	8	0.348	0.343	1.02
Montenegro Yugoslavia	Apr. 15 1979	7.0	Albatros Hotel, Ulcinj	17	0.171	0.194	0.88
Bam Earthquake, Iran	Dec. 6 2003	6.5	Bam	10	0.799	1.242	0.64
Mexico Earthquake	Sep. 19 1985	8.1	Zihuatenejo, Guerrero Ar.	135	0.103	0.318	0.32

Table 1: Selected earthquake records.

To shed light into the workings of the inelastic structure its seismic response is compared to that of an elastic model. For analyzing the structural behavior over time a specific type of wavelet analysis was employed. Fundamentally, the wavelet analysis is defined as a correlation method, wherein one can define the wavelet transform through a correlation coefficient

$$a(t) = \int_{-\infty}^{\infty} s(\tau) w^*(\tau - t) d\tau \quad (5)$$

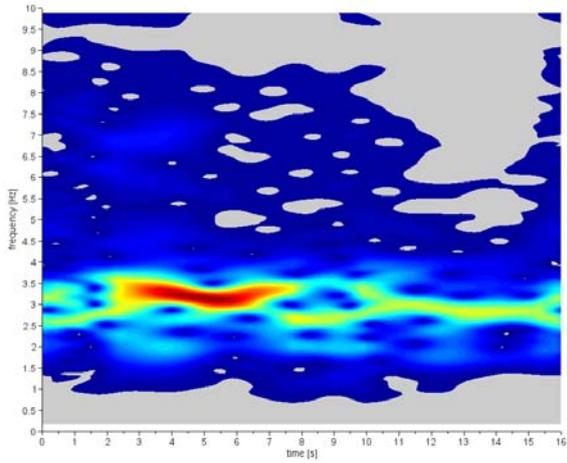
where  $w^*(t)$  is the complex conjugate to  $w(t)$ . If the signal  $s(t)$  correlates with the wavelet  $w(t)$  well the coefficient  $a(t)$  will be large; otherwise it will be small. The only requirement on the wavelet is that it be localized at a specific point of time. The harmonic wavelet developed by Newland [6] is one that is compact in the frequency domain. It has a simple structure of the form

$$W_{m,n} = \begin{cases} \frac{1}{(n-m)2\pi} & \text{for } m2\pi \leq \omega \leq n2\pi \\ 0 & \text{elsewhere} \end{cases} \quad (6)$$

where  $m$  and  $n$  are real but not necessarily integer. As one can see easily the value of the wavelet function is nonzero and constant only in a frequency band specified by  $m, n$ . The result of the procedure above is a time series corresponding to each frequency band. This time representation of a selected band is a representation of information and energy within this band. The wavelet is applied to the data in the Fourier domain which then is inversely transformed to the time domain. The harmonic wavelet transform was applied to the response of the models.

The goal of this assessment was to understand how the nonlinearity of the wall influenced its response and possibly affects its bearing capacity in an event of an earthquake. To do this, structures of different eigenfrequencies were taken for each earthquake record in such a way that the response of the model to the specific strong motion in terms of displacements is close to the defining limits of the model. That is to say, the model with the chosen natural frequency under the corresponding ground motion yields a displacement response that nears the maximum displacement values reached in the tests. This takes the model well into the nonlinear range without exceeding physically feasible boundaries. Our application of the harmonic wavelet transform produces time-frequency plots which, for the responses to the six ground motions, are given below. The figures clearly show case how the frequency response of the nonlinear structure changes over time as well as with respect to its linear counterpart. Once the structure is taken beyond its elastic limit the frequency of vibration changes significantly. The frequency drop can be sudden as in the case of El Centro or Montenegro earthquakes or more gradual like for the Honshu and Mexico earthquakes.

Harmonic wavelet transforms of the displacement responses of the elastic (left) and inelastic (right) models including time history plots of displacements for six earthquakes: Honshu, San Fernando, El Centro, Montenegro, Bam, Mexico are presented in Fig. 5-16.



Honshu 1971

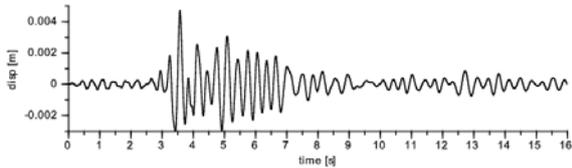


Figure 5: Honshu earthquake 1971: Displacement response of elastic model, its wavelet transform.

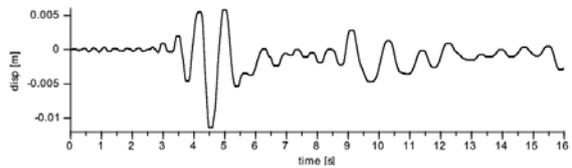
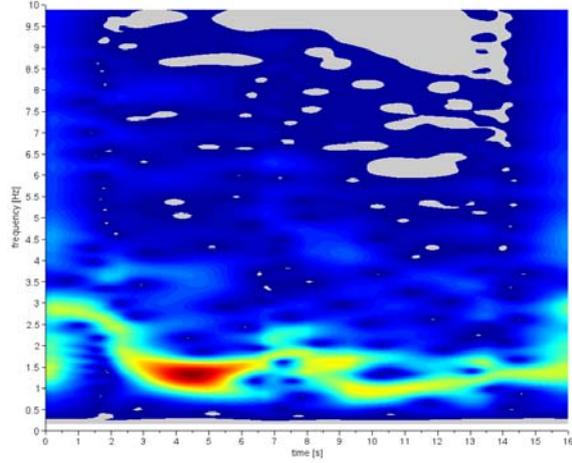
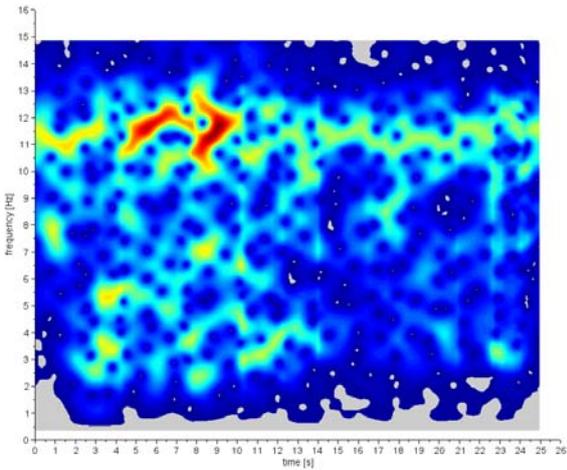


Figure 6: Honshu earthquake 1971: Displacement response of inelastic model, its wavelet transform.



San Fernando 1971

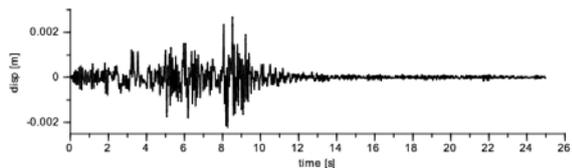


Figure 7: San Fernando earthquake 1971: Displacement response of elastic model, its wavelet transform.

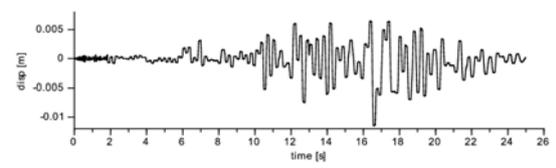
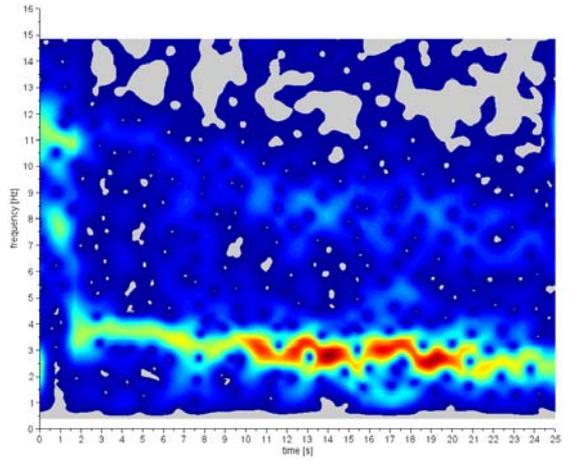
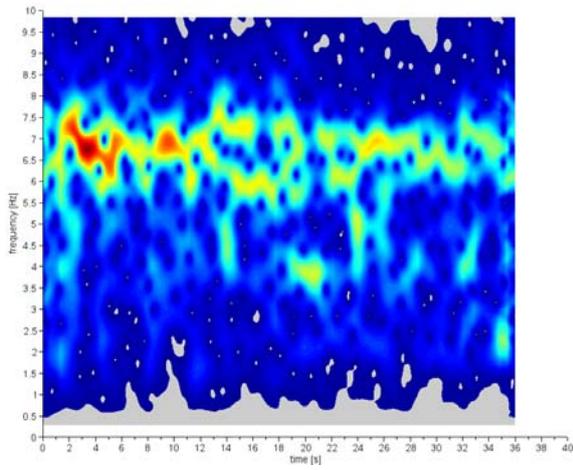


Figure 8: San Fernando earthquake 1971: Displacement response of inelastic model, its wavelet transform



El Centro 1940

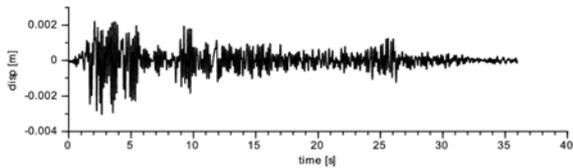


Figure 9: El Centro earthquake 1940: Displacement response of elastic model, its wavelet transform.

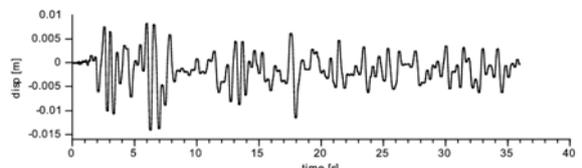
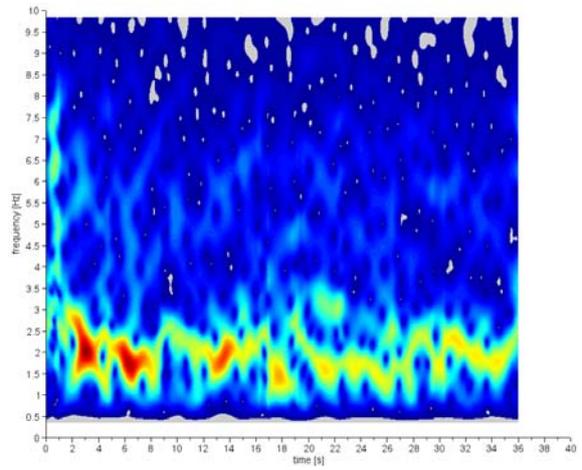
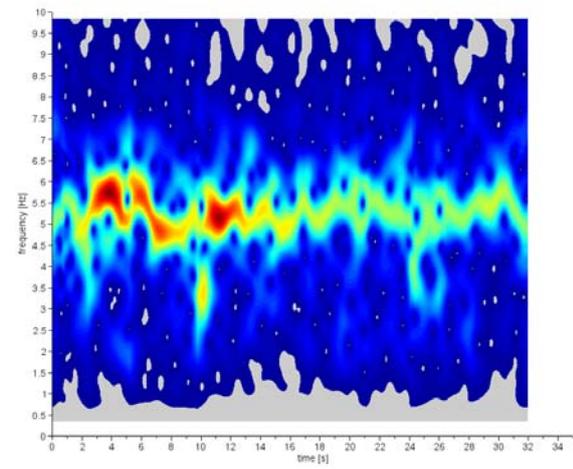


Figure 10: El Centro earthquake 1940: Displacement response of inelastic model, its wavelet transform.



Montenegro 1979

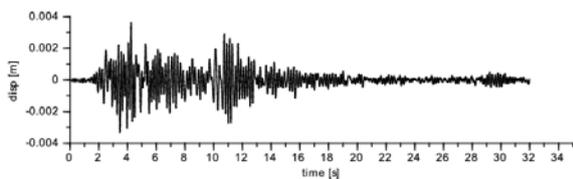


Figure 11: Montenegro earthquake 1979: Displacement response of elastic model, its wavelet transform.

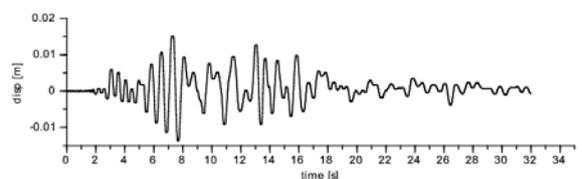
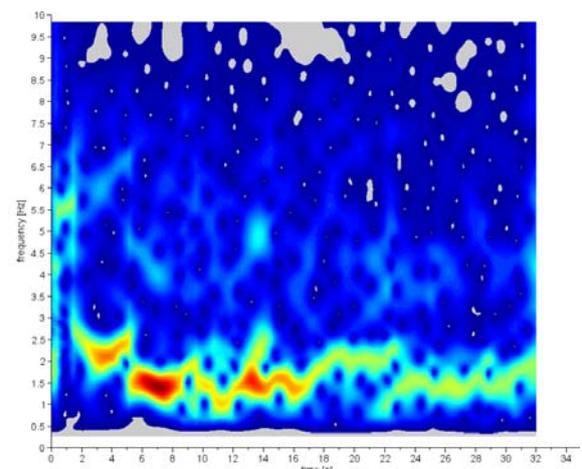
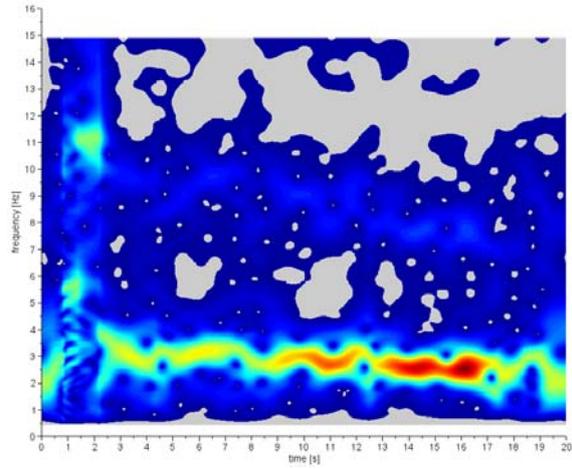
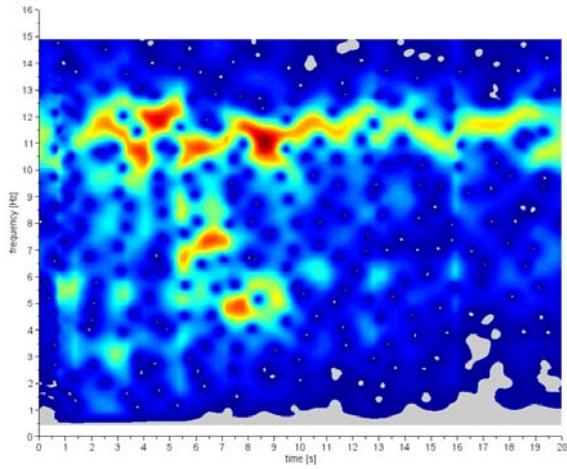


Figure 12: Montenegro earthquake 1979: Displacement response of inelastic model, its wavelet transform



Bam 2003

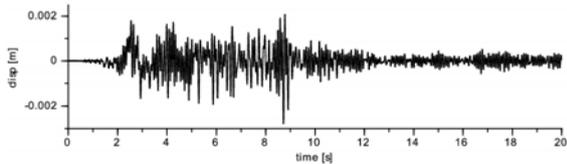


Figure 13: Bam earthquake 2003: Displacement response of elastic model, its wavelet transform.

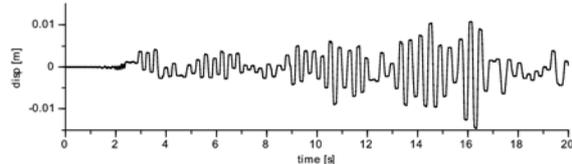
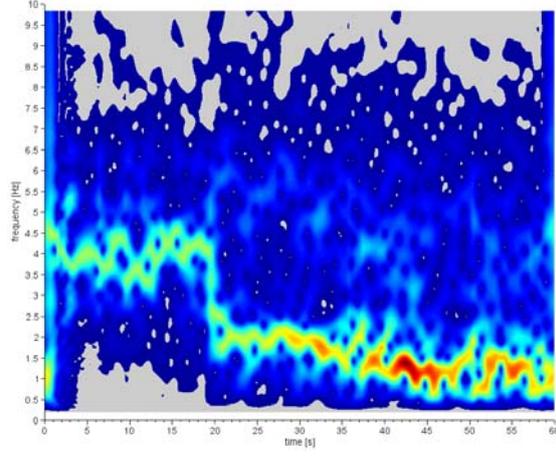
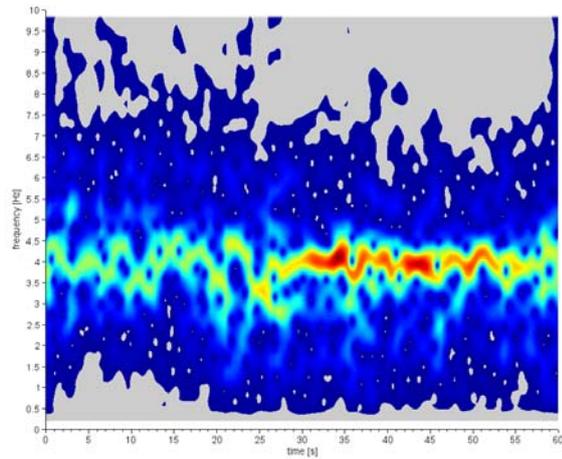


Figure 14: Bam earthquake 2003: Displacement response of inelastic model, its wavelet transform.



Mexico 1985

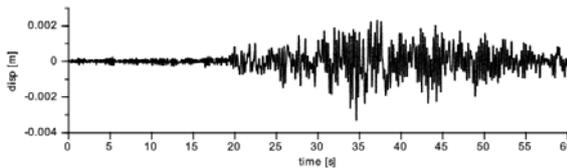


Figure 15: Mexico earthquake 1985: Displacement response of elastic model, its wavelet transform.

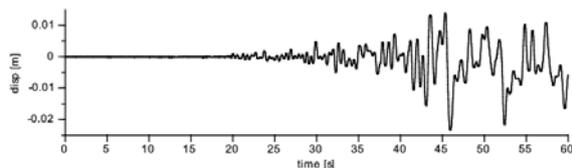


Figure 16: Mexico earthquake 1985: Displacement response of inelastic model, its wavelet transform.

## 5 NONLINEAR VS CODE BASED RESPONSE SPECTRA

To investigate possible design margins from yet another perspective we have chosen to look at nonlinear responses more specifically. The goal was to examine if the code based response spectra can be exceeded by elastic response spectra for cases when the nonlinear model still performs within the physical limits of the specimen for it was calibrated to. For the normative response spectra the ones defined in Eurocode 8 were selected. In light of this, to increase the number of European earthquakes in the set, for the scope of this assessment the accelerogram of the Friuli earthquake was added to the ones specified in Table e. Three distinct hypothetical cases were analyzed with initial periods  $T_{01} = 0.083s$ ,  $T_{02} = 0.11s$  and  $T_{03} = 0.225s$ . These periods are in relation to the initial stiffness, i.e. before cracking, of the inelastic model. All models were taken through the same procedure. For each earthquake a scaling factor was found that pushes the inelastic model to its “elastic” limit. The elastic limit of the model was selected as the point which marks the start of the yielding of the reinforcement in the physical system. This was the point of 5.4mm and 6943kN. It is noted that the point of elasticity was taken such, as it is the most correct choice for practical purposes.

Earthquake	Model 1	Model 2	Model 3	Model 0
	$T_{01} = 0.083s$	$T_{02} = 0.11s$	$T_{03} = 0.225s$	$T_0 = 0.027s$
Honshu 1971	3.44	3.17	1.33	57.5
San Fernando 1971	0.71	0.38	0.09	7.9
El Centro 1940	2.00	1.12	0.28	23.0
Montenegro 1979	3.12	1.61	0.43	47.5
Bam 2003	0.63	0.34	0.19	9.1
Mexico 1985	7.11	4.05	0.64	76.0
Friuli 1976	2.31	1.14	0.29	21.5

Table 2: Example Scaling factors for earthquake accelerograms that take the model to its elastic limit.

Elastic response spectra derived from scaled accelerograms were compared to design response spectra from EC8 for model 1, model 2 and model 3 in Fig. 17-19.

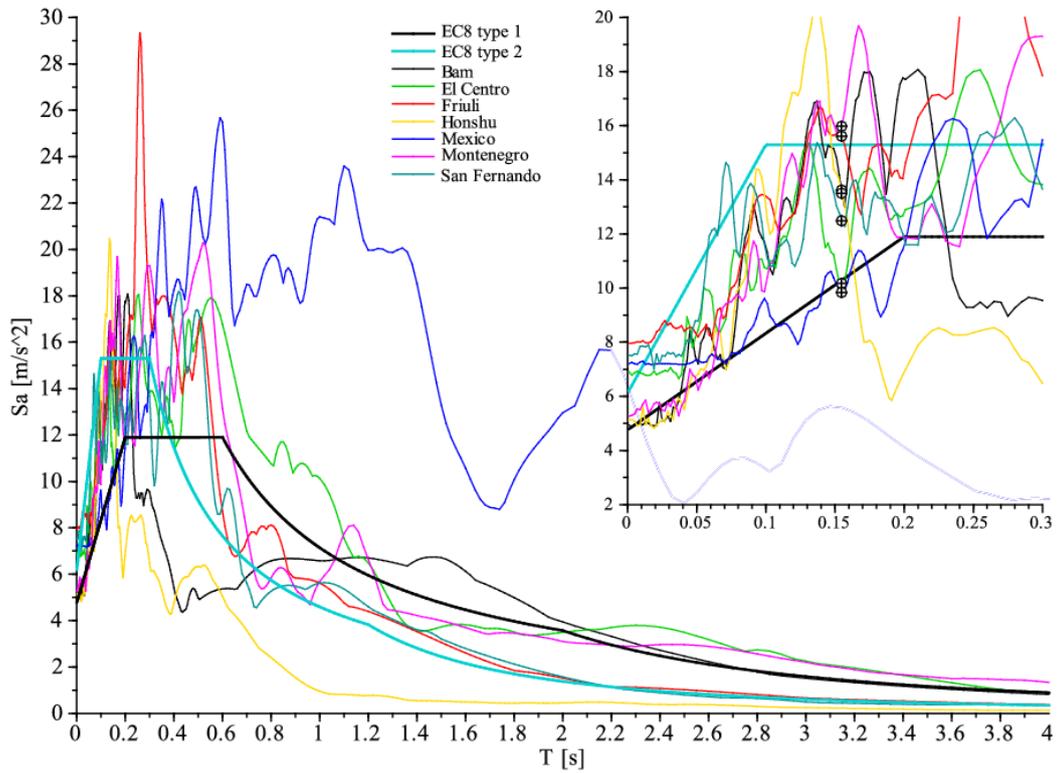


Figure 17: Elastic response spectra derived from scaled accelerograms compared to design response spectra from EC8 for model 1.

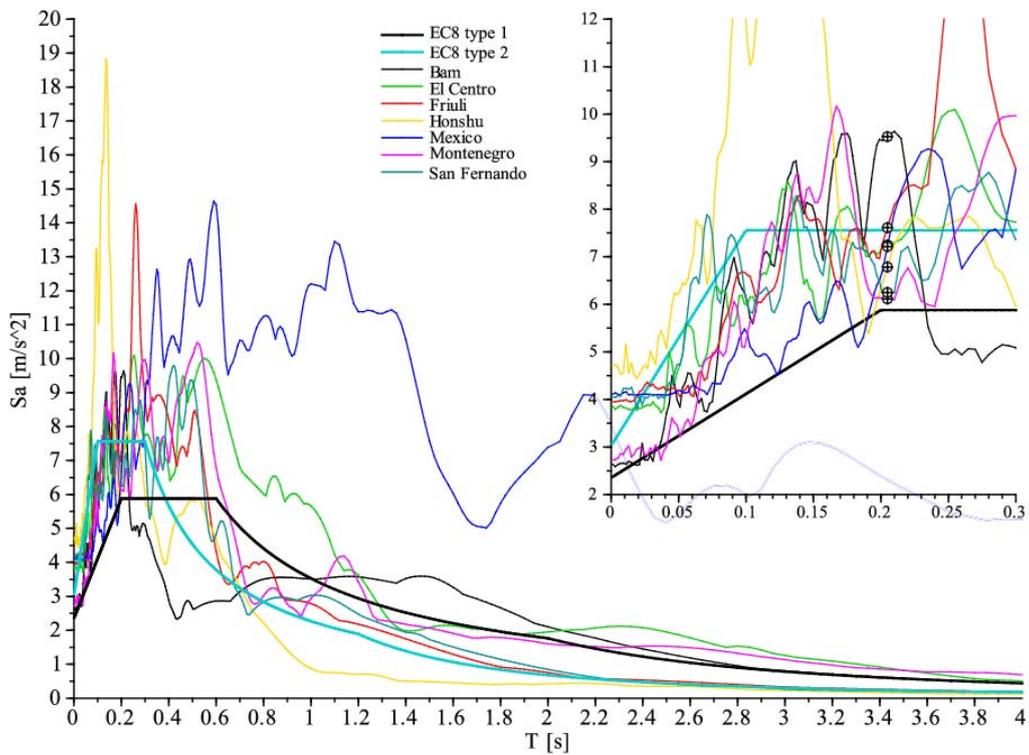


Figure 18: Elastic response spectra derived from scaled accelerograms compared to design response spectra from EC8 for model 2.

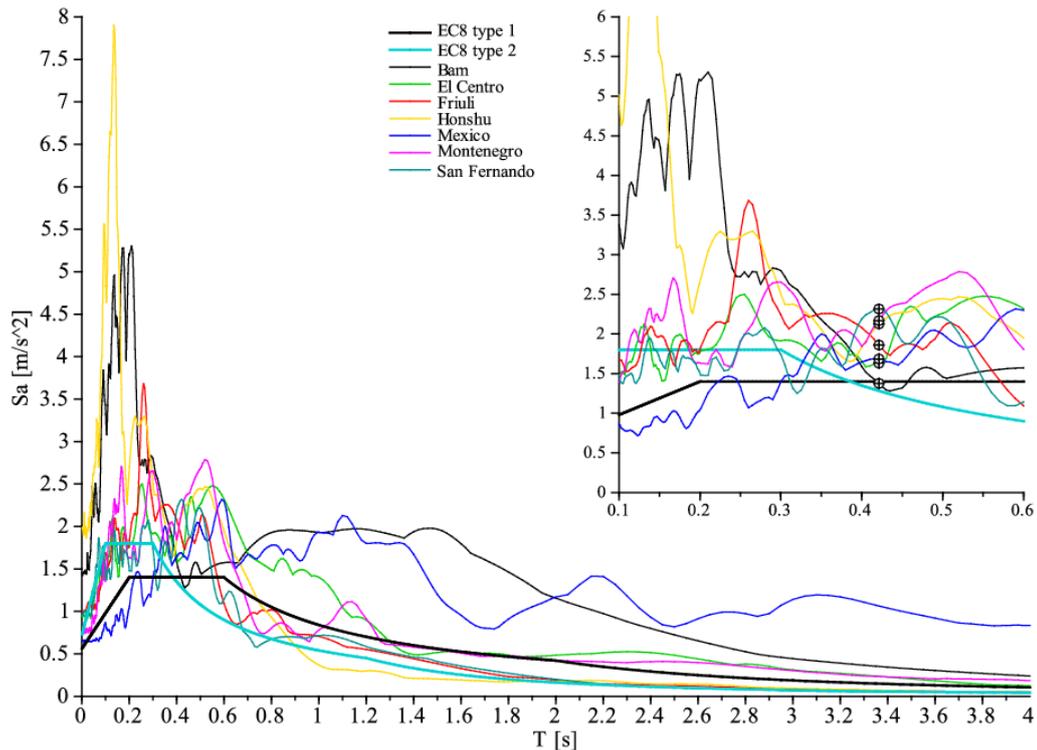


Figure 19: Elastic response spectra derived from scaled accelerograms compared to design response spectra from EC8 for model 3.

To define the imagined design capacity of the wall the value of 8.0MN, derived from testing, was assumed. To simulate design of the wall to this resistance the design response spectra needed to be scaled in such a way that at the design period they yield a design seismic force of 8.0 MN. This was simply achieved by an appropriate choice of base ground acceleration  $a_g$  for the type 1 response spectrum from EN 1998. Type 2 response spectrum was also plotted for reference. Viewing the results it is very evident that the model earthquake response spectra do reach above the design spectra. In some cases, especially for low periods, the overshoot is significant. Still when considering the differences at the design period (marked with dots in the subplots) many of the model response spectra lie above the design spectrum. In these cases it is not considered substantial, possibly with some exceptions. The interpretation of the results is that the linear response spectrum computed for a scaled excitation, which takes the nonlinear model to its elastic limit, yields forces that are higher than those defined by the code based design spectrum. Thus a reinforced concrete structure, namely a shear wall, to the extent it is represented by the mathematical model, can bear within its linear limits, dynamic forces higher than designed for based on a selected code, EN 1998 in this case.

## 6 CONCLUSIONS

- Based in the test data a mathematical hysteretic model was developed based on Litton's modification of the original Takeda model. This is a single degree of freedom nonlinear material model with the feature of pinching included. The model is tuned by two sets of four steering parameters for the two opposite quadrants. Upon the observation that the model would require different steering parameters for small and large displacement a functional relation was introduced for them. With such a refined model a good agreement with test data was reached.

- The behavior of a massive shear wall, represented by the mathematical model was analyzed for different selected earthquake records. The harmonic wavelet transform was applied to unravel the frequency response of the modelled structure over the time of excitation. The frequency shift could be evidently shown with the point of transition to inelastic yielding visible. For this analysis models of different natural frequencies were chosen for each earthquake record, so that they are driven by the excitation close to the maximum displacement value of the test specimen the model was based on. This was needed to study the inelastic behavior during an earthquake.
- The presented investigations were triggered by the need to explain seemingly unexpected phenomena observed in the wake of some earthquakes in nuclear power plants. And although nuclear facilities are designed to very strict criteria and substantial margins, there are indications of intrinsic margins beyond the intended ones due to disparities between the physics involved and the simplifications in the design procedures. It comes as no surprise that the accepted design procedures applicable in cases of massive shear walls were shown to be conservative.

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