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NUMERICAL MODELS FOR SIMULATING THE DYNAMIC BEHAVIOUR OF FREESTANDING ANCIENT COLUMNS

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Abstract

In this contribution, three numerical models are considered and compared, with the main purpose of simulating the dynamic behavior of monolithic and multi-drum freestanding ancient stone columns. The behavior of such classical and historic structural elements, typical of the Mediterranean area and that are frequently subject to seismic actions, is characterized by a strong nonlinearity due to sliding and rocking.
A simple and effective rigid beam model, able to numerically solve the equations of motion of the column also accounting for Housner's hypotheses, is introduced for first and validated with respect to a software based on the Discrete Element Method (DEM), which has already proven its effectiveness in representing the behavior of columns and, more generally, masonry structures. Furthermore, a rigid block model accounting for the nonlinear behavior of the interfaces between the blocks is considered. On one hand, the rigid models can represent columns behavior with a not significant computational effort; on the other hand, the DEM is able to better describe the strong nonlinearity of columns behavior, with the detection of new contacts and the results in terms of collapse mechanisms characterized by large displacements that may be experienced by the blocks during the dynamic excitations.
In this contribution several preliminary comparisons between the models are carried on by considering a multi-drum column and an equivalent monolithic one subject to a set of harmonic excitations with varying input frequency and acceleration amplitude.

Keywords: Masonry, Columns, DEM, Rigid Beam Model, Dynamic response.

1 INTRODUCTION

Monolithic and multi-drum stone columns are structural elements typical of ancient temples that can be found in the Mediterranean area, thanks to the diffusion of Greek and Roman civilizations. In the original constructions, columns were connected by top beams in order to transfer vertical loads from the top roof to the ground; however, due to damage caused by historical events and seismic events that struck the Mediterranean regions during their life, many columns are now free-standing and more prone to collapse for earthquake actions.

Starting from the last century and until the current days, the analytical and numerical assessment of monolithic and multi-drum column behavior has received particular attention. The pioneering analytical model proposed by Housner [5] studied the behavior and the possible overturning of a single rigid block subjected to horizontal excitations. Further research activities focused on studying the behavior of monolithic elements both numerically and experimentally; as well as on the behavior of columns made of multi-drums (e.g. [1, 8, 10]).

In these studies, the Discrete Element Method (DEM), initially introduced for the analysis of rock masses [7]. The advantage of the approach lies in its possibility to simulate each drum as an independent body, which can be subject to large displacements during an excitation. According to [1], although three-dimensional analysis can provide a more realistic and accurate representation of the dynamic behavior of ancient columns, two-dimensional analysis can still be performed using DEM since they are more time efficient and less sensitive to the contact parameters

In this work, a new simple and effective numerical model for studying the dynamic behavior of both monolithic and multi-drum columns is proposed. The model considers a multi-drum column as an assemblage of vertically aligned rigid beam elements, where each drum is represented by a beam element and each contact is represented by a node. The dynamic equilibrium equation of the system is solved by means of a Runge-Kutta solver for ordinary differential equations accounting for a nonlinear moment-rotation law at each joint and without considering sliding failure.

The accuracy and effectiveness of the proposed model is validated by comparing its numerical results with those given by the two-dimensional DEM code UDEC [6]. A series of dynamic analyses were performed in which monolithic and multi-drum ancient columns subjected to different in frequency and amplitude harmonic excitations, as per [10]. Furthermore, a simpler comparison between the proposed rigid beam model and a rigid block model undertaken, for evaluating the influence of the no-sliding hypothesis in the numerical results.

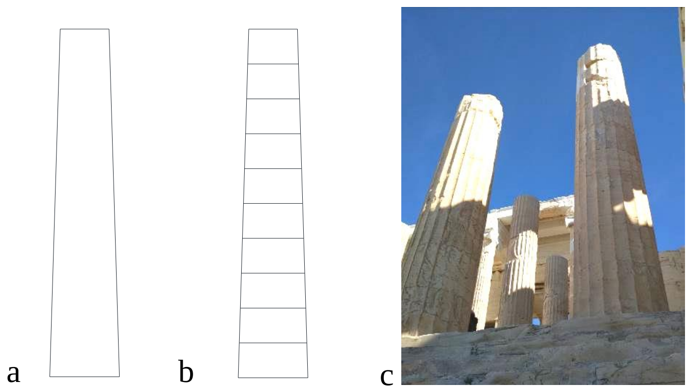

Figure 1: Scheme of monolithic (a) and multi-drum (b) columns; freestanding columns in Athens Acropolis (c).

2 NUMERICAL MODELS

As stated in the introduction, three numerical models are considered for studying the behavior of a monolithic column (Fig. 1a) and of an equivalent in size multi-drum column (Fig. 1b). The models aim to simulate the behavior of free-standing columns typical of ancient Greek and Roman temples (Fig. 1c).

2.1 Rigid Beam Model

Let us consider a generic multi-drum column composed of n drums. Then a rigid beam model can be developed which is composed of n beam elements and n+1 nodes as shown in Fig. 2a, b. Each beam element represents a drum of the column and each node represents an interface between the drums. In particular, the first node/interface represents the contact between the ground and the first drum. Since the aim of this work is the assessment of columns behaviour subject to horizontal excitation, only horizontal translational degrees of freedom are considered, namely u_i, $\dot{u}_i$, and $\ddot{u}_i$ represent, respectively, nodal horizontal translation, velocity and acceleration. It is worth noting that the proposed model is also able to describe the behavior of a monolithic column by setting n = 1.

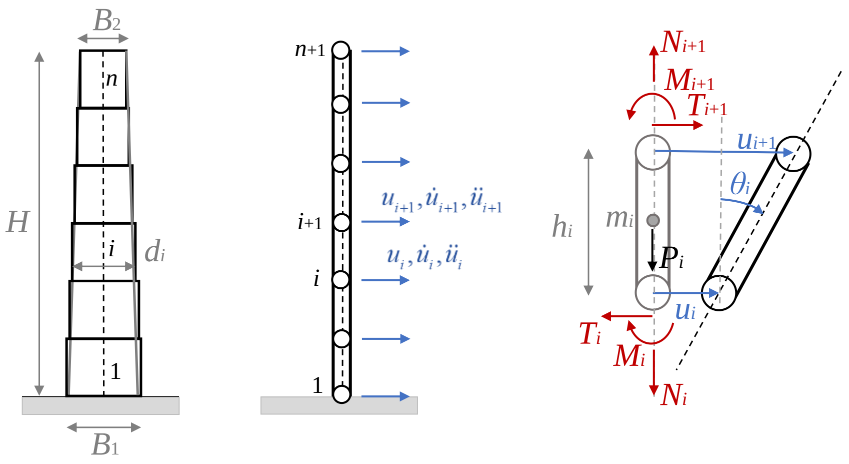

Figure 2: Multi-drum column (a), corresponding rigid beam model (b), generic beam element (c).

Each i-th beam element is characterized by a mass m_i, which depends on material density and on the volume of the corresponding drum, which is considered for simplicity as a rectangular prism having an average width with respect to upper and lower drum width (Fig. 2a). Furthermore, due to the rigid beam hypothesis, each element is subject to a rigid rotation depending on the horizontal end translations and beam height h_i (Fig. 2c):

$$\theta_i = \frac{u_{i+1} - u_i}{h_i} \tag{1}$$

Internal forces of the beam are given by a shear force T_i, and a bending moment M_i, acting at each beam end (Fig. 2c); a normal force N_i is also present. The translational and rotational equations of motion for a beam element may be written as follows:

$$\begin{aligned} \frac{\ddot{u}_{i+1} + \ddot{u}_i}{2} m_i &= T_{i+1} - T_i \\ \ddot{\theta}_i I_{Gi} &= -M_{i+1} + M_i + T_{i+1}\frac{h_i}{2} + T_i\frac{h_i}{2} \end{aligned} \tag{2}$$

Where I_{Gi} is the polar inertia of the drum corresponding to the beam element. For simplicity, the static equilibrium in vertical direction, which allows to define normal forces as function of the gravity loads P_i, is not written in detail. Considering a column subject to a horizontal ground acceleration $a_g(t)$, equations of motion (2) are subject to the following boundary conditions at column base and top nodes:

$$\begin{aligned} \ddot{u}_1 = \ddot{u}_g &= a_g(t) \\ T_{n+1} &= 0 \\ M_{n+1} &= 0 \end{aligned} \tag{3}$$

that allow to write in matrix compact form the two set of equations of motion (2) for the entire multi-drum column:

$$\begin{aligned} \mathbf{T} &= \mathbf{M}_a\ddot{\mathbf{u}} - \mathbf{A}_g \\ \mathbf{M} &= \mathbf{G}\mathbf{T} + \mathbf{I}_G\ddot{\mathbf{u}} - \mathbf{B}_g \end{aligned} \tag{4}$$

In the equations above, vectors **T** and **M** collect shear forces and bending moments from node i to n: $\mathbf{T} = [T_1\ T_2\ \ldots\ T_n]^T$, $\mathbf{M} = [M_1\ M_2\ \ldots\ M_n]^T$; whereas vector $\ddot{\mathbf{u}}$ collects horizontal accelerations from node 2 to n+1: $\ddot{\mathbf{u}} = [\ddot{u}_2\ \ddot{u}_3 \ldots \ddot{u}_{n+1}]^T$. Matrices $\mathbf{M}_a$, $\mathbf{G}$, and $\mathbf{I}_G$ can be called, respectively, mass coefficient matrix, geometric coefficient matrix, and polar inertia coefficient matrix. Vector $\mathbf{A}_g$ is characterized by null values except the first component, representing the acceleration at the base of the column: $\mathbf{A}_g = [a_g(t)m_1/2\ 0\ \ldots\ 0\ 0]^T$; and, similarly, $\mathbf{B}_g = [a_g(t)I_{G1}/h_1\ 0\ \ldots\ 0\ 0]^T$. Substituting the first of Eq. (4) into the second one, the system of differential equations to be solved for obtaining the displacements of the multi-drum column is:

$$\mathbf{M}(\boldsymbol{\theta}) = \mathbf{G}\mathbf{M}_a\ddot{\mathbf{u}} - \mathbf{G}\mathbf{A}_g + \mathbf{I}_G\ddot{\mathbf{u}} - \mathbf{B}_g \tag{5}$$

Where each bending moment M_i in **M** depends on the rotation θ_i of the corresponding i-th drum (Eq. 1). The system of differential equations in (5) is solved by means of a Runge-Kutta ODE solver. At this stage, the nonlinear behavior that can affect the multi-drum column is the rocking phenomenon at each interface between the drums, whereas, following Housner's hypothesis, shear failure can not occur. The bending moment M_i at each interface must follow a bi- or tri-linear moment-rotation relationship, which represents the maximum stabilizing moment for varying block rotation (Fig. 3), and it is slightly modified with respect to Housner's law by means of an initial elastic stiffness $K_{M,i}$ and a smoothing parameter $\xi \leq 1$.

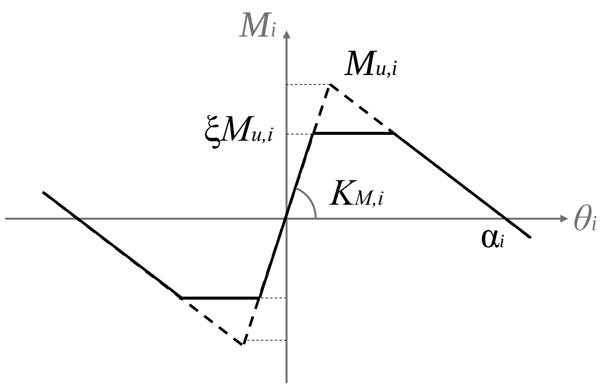

Figure 3: Moment-rotation relationship.

The maximum stabilizing moment in Fig. 3 is given by:

$$M_{u,i} = \frac{d_i}{2} \sum_{j=i}^{n} m_j \cdot g = \frac{d_i}{2} \sum_{j=i}^{n} P_j \tag{6}$$

where d_i is the width of the *i*-th drum, hence $d_i/2$ represents the maximum eccentricity of the normal force at the *i*-th joint. The angle α_i represents the critical angle of the generic drum [5].

2.2 Discrete Element Model

The DEM and, in particular, the computer code UDEC [6], have been successfully adopted by several researchers to study the dynamic behavior of monolithic and multi-drum columns [1, 3]. With UDEC, blocks can be modelled as rigid or deformable blocks. However, model movements are mainly given by the relative displacements between the blocks, rather than single block deformability. Blocks interact together by means of contact points, and the contacts between the blocks are continuously detected during a dynamic analysis. Contact behavior is characterized by normal and shear forces, f_n and f_s, with respect to contact surface orientation. Such forces in the elastic range are governed by a normal and a shear stiffness:

$$\begin{aligned} \Delta f_n &= -k_n \Delta u_n A_c \\ \Delta f_s &= -k_s \Delta u_s A_c \end{aligned} \tag{7}$$

where Δ denotes the increment of forces and relative normal and shear displacements (u_n, u_s) and A_c is the contact area. Assuming dry joints, only compressive normal forces are admitted, and the maximum shear force depend on the friction angle φ of the contact surface: $f_{s,\max} = f_n \tan(\varphi)$.

2.3 Rigid Block Model

A rigid block model was also adopted for performing further numerical comparisons with the first two models. Rigid block models were introduced for representing in- and out-of-plane elastic behavior of one-leaf masonry panels [11], and it has been extended accounting for material nonlinearity [12]. The model assumes blocks as rigid bodies; hence degrees of freedom are given by block center translations and block rigid rotation with respect to each center. The joints between the blocks are modelled as elastic-plastic one dimensional interfaces. Incremental interface actions, namely a normal and a shear force, (ΔF_n, ΔF_s), and a bending moment ΔM, depend on block relative displacements (Δu_n, Δu_s) and relative rotation $\Delta\theta$ and follow a Mohr-Coulomb yield criterion characterized, in this case, by a null cohesion and the same friction angle φ adopted with the DEM. Normal and shear elastic stiffness parameters are assumed equal to those of the DEM multiplied for the entire joint area A, whereas the bending stiffness is equal to the normal stiffness multiplied for joint moment of inertia I:

$$\begin{aligned} \Delta F_n &= K_n \Delta u_n = (k_n A) \Delta u_n \\ \Delta F_s &= K_s \Delta u_s = (k_s A) \Delta u_s \\ \Delta M &= K_r \Delta\theta = (k_n I) \Delta\theta \end{aligned} \tag{8}$$

It is worth noting that interface actions can be directly compared with the nodal forces of the rigid beam model. Furthermore, the bending stiffness and the moment-relative rotation law can be assumed coincident with those adopted with the rigid beam model, (Fig. 3). Recent developments of this model also allow to consider generic quadrilateral or polygonal rigid elements [13]. Therefore, the actual trapezoidal section of the monolithic column and of each drum can be taken into account.

3 NUMERICAL TESTS

A parametric investigation of the response of the columns subjected to harmonic excitations in horizontal direction was performed. The case studies already considered in the recent contribution by Sarhosis et al. [10] were chosen. In particular, the monolithic column has a height (H) equal to 5 m, base width (B_1) equal to 0.96 m and top width (B_2) equal to 0.66 m. The multi-drum column has the same overall dimensions of the monolithic one and it is composed of 12 equally spaced drums. Masonry density was taken as 1600 kg/m^3 and the block elastic modulus as 2500 MPa. Also, the joint normal and shear stiffness was equal to 5×10^{10} and 2.5×10^{10} N/m^3, respectively. Results recently obtained with DEM are given by a set of 30 different dynamic analyses with frequency varying from 0.66 to 4 Hz and base acceleration amplitude (a) varying from 0.1 g to 0.5 g. The rigid beam model allows to perform a larger set of analyses, in order to obtain a more accurate safe-unsafe domain for both column types. The rigid block model used herein to performing pushover analyses with forces equivalent to the horizontal excitation.

3.1 Monolithic column subject to harmonic horizontal excitation

Initially, numerical simulations were performed to assess the dynamic behavior of the monolithic column. Figs. 4 and 5 compare the base and top column displacements obtained with the rigid beam model and with the DEM model, respectively, for several acceleration amplitude and frequency values. Top displacements turn out to be in quite good agreement between the two models, even if the rigid beam model already shows a collapse with 0.66 Hz and 0.2 g. Base displacements obtained with the rigid beam model were generally larger than those obtained with the DEM model. Collapse mechanisms for this column type are not represented for simplicity, given that they are characterized by a simple overturning of the column.

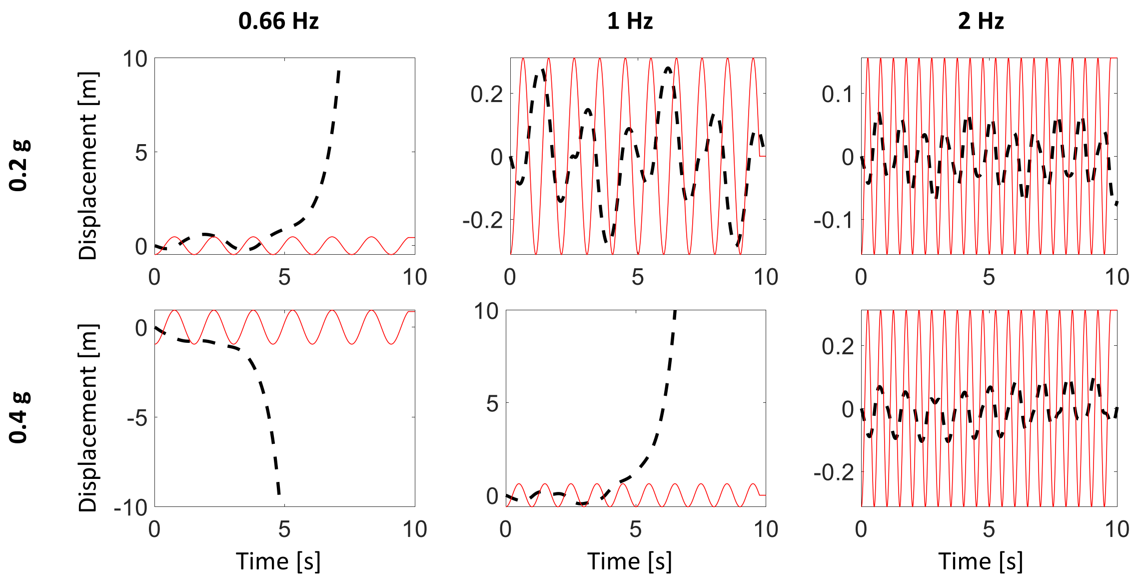

Figure 4: Base (red continuous line) and top (black dashed line) displacements for a monolithic column subject to several harmonic excitations. Rigid beam model results.

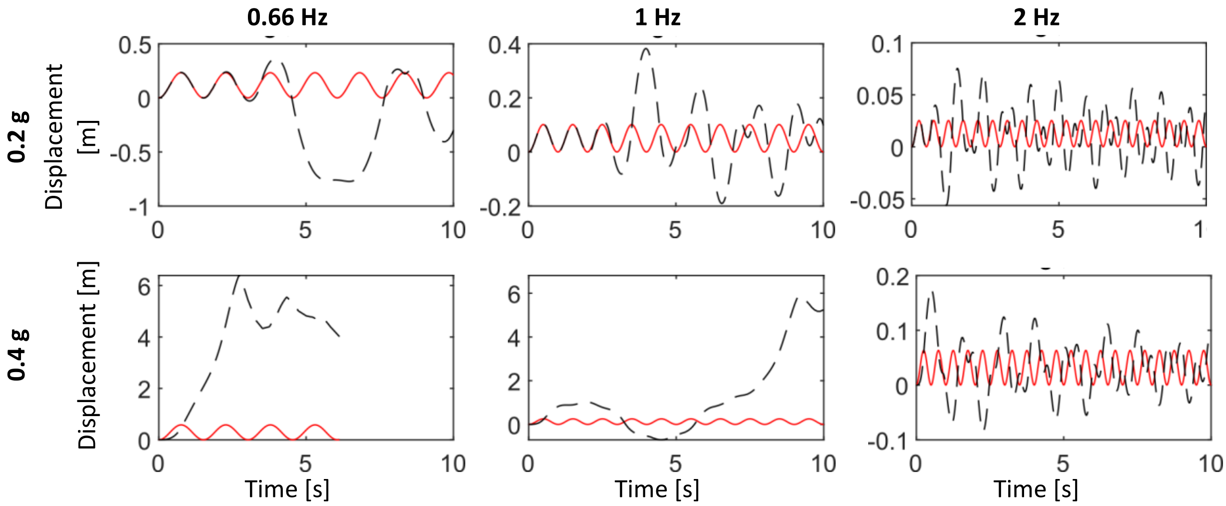

Figure 5: Base (red continuous line) and top (black dashed line) displacements for a monolithic column subject to several harmonic excitations. DEM (UDEC) results.

Fig. 6 compares the safe-unsafe domains obtained with the rigid beam model and the DEM model. Both domains show a collapse acceleration that increases for increasing input frequency. The small computational effort required by the rigid beam model allows to obtain a more accurate domain, with smaller collapse acceleration amplitudes with respect to the DEM for small input frequency values. However, further developments of this work will also try to define a more detailed safe-unsafe domain by means of the DEM. For decreasing input frequency, the collapse acceleration given by the rigid beam model is close to 0.16 g, slightly smaller than the collapse acceleration obtained with the rigid block model, equal to 0.205 g. Such difference is due to the approximated rectangular column shape assumed by the rigid beam model.

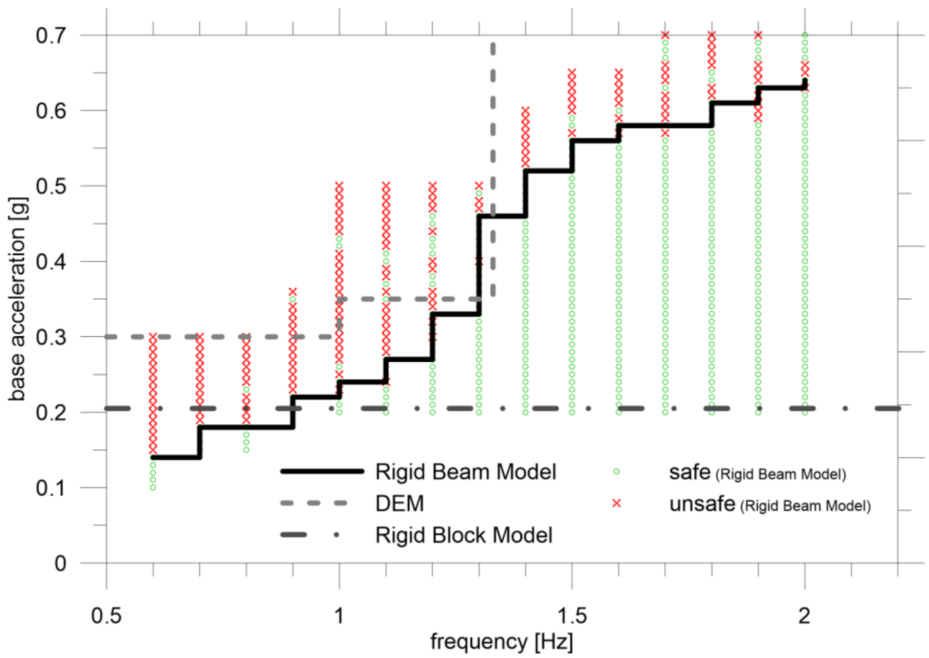

Figure 6: Safe-unsafe domain for a monolithic column subject to harmonic excitations. Rigid beam model results (symbols and continuous line), DEM (UDEC) results (dashed line).

3.2 Multi-drum column subject to harmonic horizontal excitation

A comparison of the dynamic behavior of the multi-drum column was also undertaken. The multi-drum column was subjected to different in amplitude and frequency harmonic loads. Figs. 7 and 8 present the base and top column displacements determined with the rigid beam model and with the DEM, respectively, for several acceleration amplitudes and input frequencies. Base displacements obtained with the rigid beam model are generally larger than those obtained with the DEM model. In this case, it was found that the multi drum column analyzed using the rigid beam model is more prone to collapse with respect to the one analyzed using by the DEM model.

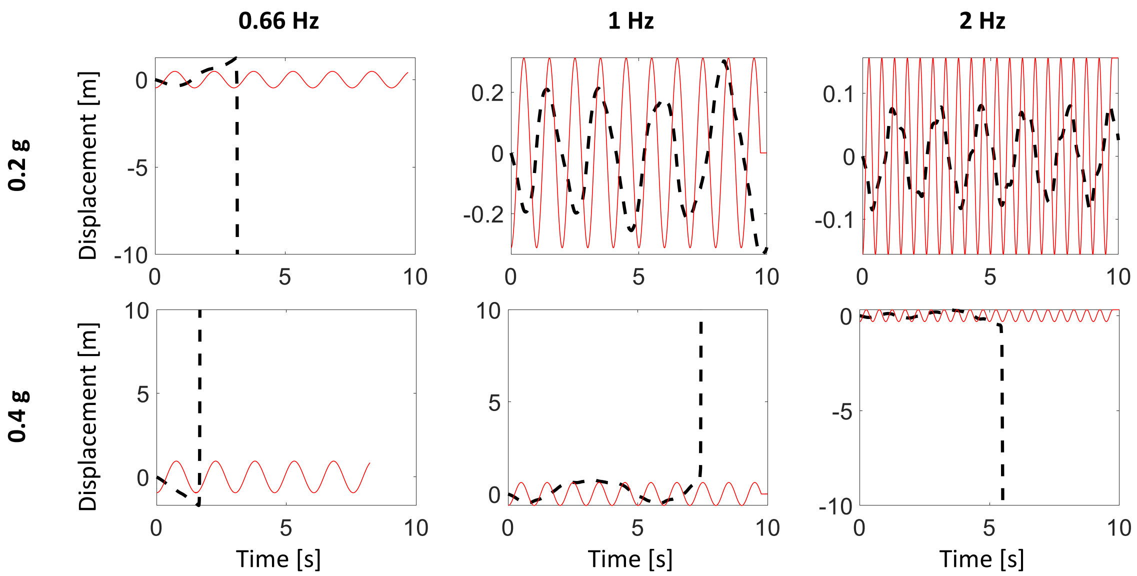

Figure 7: Base (red continuous line) and top (black dashed line) displacements for a multi-drum column subject to several harmonic excitations. Rigid beam model results.

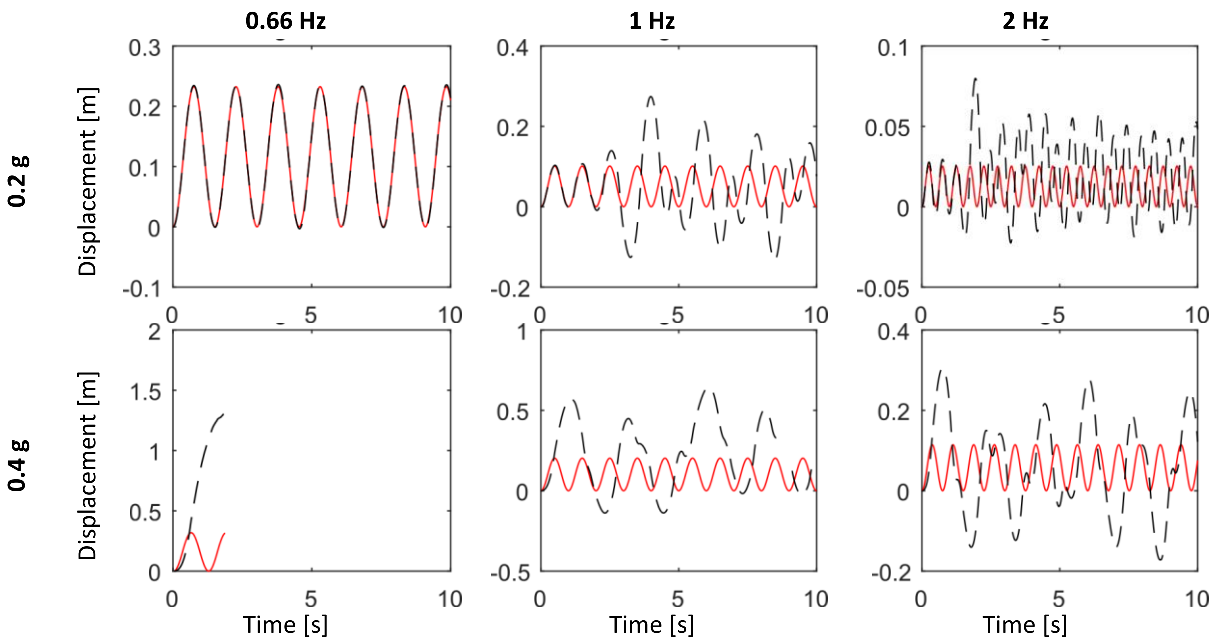

Figure 8: Base (red continuous line) and top (black dashed line) displacements for a multi-drum column subject to several harmonic excitations. DEM (UDEC) results.

The safe-unsafe domain of the multi-drum column modelled by rigid beams is shown in Fig. 9. For the development of the safe-unsafe graph, some DEM results were obtained from [10] in which the authors have carried out the analyses. Although there is a difference in the results obtain between the rigid block model and the DEM model, it shows that the multi-drum column modelled by rigid beams is subject to collapse for accelerations up to 0.2 g for frequencies less than 1 Hz. In addition, for increasing input frequency, the collapse acceleration slightly increases up to 0.36 g with 2 Hz. The collapse accelerations turn out to be smaller than those obtained with the DEM. Such a difference is mainly given by the effect of sliding, which is neglected by the rigid beam model. The pushover analysis with the rigid block model is characterized by a collapse acceleration equal to that obtained in the previous case, accordingly to the well-known hypothesis of monolithic behavior of a multi-drum column subject to static actions [14].
Focusing only on the proposed rigid beam model and comparing the safe-unsafe domains obtained for the monolithic column and the equivalent multi-drum one, the former case turns out to be less prone to collapse with respect to the latter case. This aspect is not in agreement with the information obtained with the column types modelled by DEM, which are characterized by a smaller unsafe region of the domain with the multi-drum column.

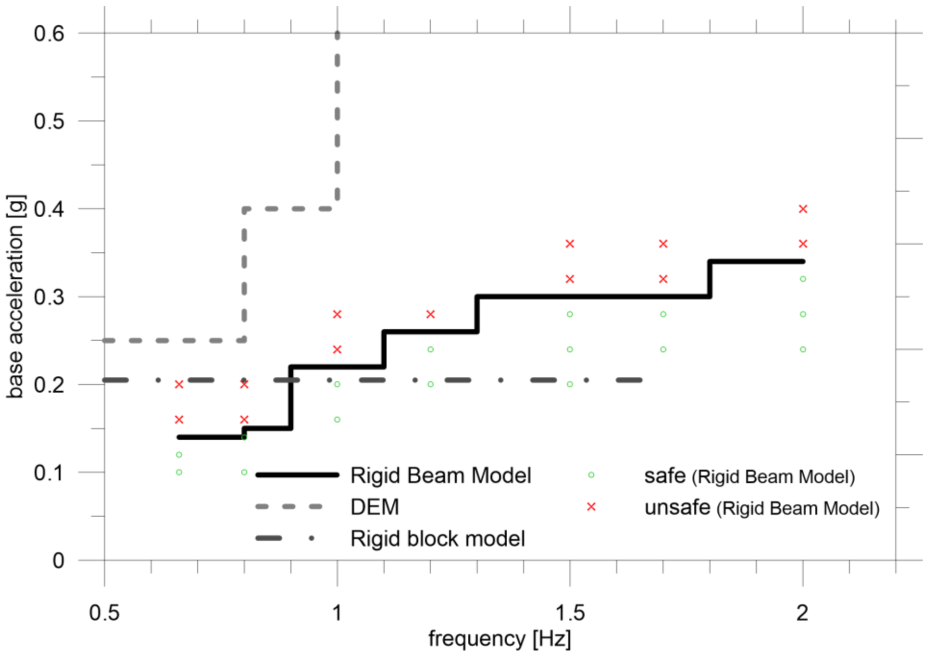

Figure 9: Safe-unsafe domain for a multi-drum column subject to harmonic excitations. Rigid beam model results (dashed line), DEM (UDEC) results (continuous).

4 CONCLUSIONS

A simple and effective rigid beam model for studying the dynamic behavior of freestanding monolithic and multi-drum columns has been proposed. The model assumes each drum as a rigid beam element and each joint between the drums as a node able to move horizontally. The nonlinear behavior of the model is obtained by setting a bi or tri-linear moment-rotation nonlinear law. The rigid beam model turned out to be in sufficient agreement with the DEM assumed as reference for performing harmonic tests with varying acceleration amplitude and input frequency. The rigid beam model is characterized by larger base displacements with respect to the DEM model and by a smaller safe domain for both column types, with small col-

lapse acceleration values for decreasing frequency with respect to the DEM model. Also, the rigid block model, even if it has been adopted for simple pushover tests, shows an intermediate behavior between the DEM and the rigid beam model. In further developments of this contribution, harmonic tests will be performed also with the rigid block model. It is worth noting that the proposed analyses were limited to the 2D case. In particular, the 2D DEM model considered in this study assumes a unitary thickness of the column, leading to a rectangular column cross-section. The proposed rigid beam model and the rigid block model, instead, are able to account for the actual circular cross-section of the column, leading to more accurate results that can be compared with more complex 3D analyses.
Further developments of the proposed rigid beam model will regard the use of real ground motions for performing numerical tests on monolithic and multi-drum columns. Results will be compared with existing numerical and laboratory results. Further developments of the proposed rigid beam model will take into account the possible sliding between the drums, by assuming, for instance, a Mohr-Coulomb frictional law for restraining nodal shear actions and by allowing relative horizontal displacements at joint level.

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